ECONOMICS AND FINANCE OF PENSIONS

Lecture 3

HOUSEHOLD PORTFOLIOS

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Introduction

•The last lecture, we built our first lifecycle models and discussed how we might incorporate different features of individuals into them:

- Human capital
- Residential housing
- Financial assets
- Mortality
- Pensions

Introduction

- In this lecture we are going to use these models to examine household *asset allocation* from an empirical and a theoretical perspective

- We will cover a combination of:
- Empirical work (looking at actual HH portfolios and how they change over time/age/country)
- Theory (looking at the different types of lifecycle models in discrete and continuous time)
- … but we will start with the theory

Outline

- Portfolio theory
  - Markowitz
  - Samuelson
  - Merton
  - Add-ons
- Empirical household portfolios
  - Three stylised facts
  - Pensions and housing
- How good is the theory?
- Extensions of the theory

Markowitz

- You will all be familiar with Markowitz portfolio theory
  - It’s a static model
  - Nearly 60 years old
  - Based on means and variances
  - Assumes normal distributions
  - And it’s still actually used in practice!

Samuelson and Merton

- We've already seen Samuelson's OLG model, the extension of which is in your textbook
- In 1969, he and Robert Merton published two companion papers in the Review of Economics and Statistics
- They both dealt with the same problem: the joint economic determination of optimal consumption and optimal asset allocation decisions in a dynamic framework
- Samuelson's paper used stochastic dynamic programming to solve the problem; Merton's used (continuous-time) stochastic optimal control
- Both papers are on WebCT

Samuelson's model

- Agent wishes to maximise the discounted utility of consumption as before:

$$\max_{\{c_0,\ldots,c_n,\alpha_0,\ldots,\alpha_n\}} E_0 \sum_{i=0}^{T} \rho^i u(c_i)$$

- But now, agent can choose both consumption and the asset mix at each point in time:

$$w_{i+1} = \tilde{R}_{i+1}(w_i - c_i)$$

$$\tilde{R}_{i+1} = \alpha_i \tilde{\mu} + (1-\alpha_i)(1+r) = \alpha_i(\tilde{\mu} - 1 - r) + (1+r)$$

- α_i is the proportion in risky assets at time *i*
- How do we solve this model?

It's easy:

Write out the value function:

$$V_i(w_i) = \max_{\{c_i,\ldots,c_n,\alpha_i,\ldots,\alpha_n\}} E_i \sum_{j=i}^{n} \rho^{j-i} u(c_j)$$

Write out the Bellman equation:

$$V_i(w_i) = \max_{\{c_i,\alpha_i\}} u(c_i) + \rho E_i[V_{i+1}(\tilde{w}_{i+1})]$$

Take F.O.C's w.r.t. to c:

$$u'(c_i) = \rho E_i[\tilde{R}_{i+1} V_{i+1}^{'}((w_i - c_i)\tilde{R}_{i+1})]$$

Use the envelope theorem:

$$V_i'(w_i) = \rho E_i[\tilde{R}_{i+1} V_{i+1}^{'}(\tilde{w}_{i+1})] \quad \text{so} \quad V_{i+1}'(w_{i+1}) = u'(c_{i+1})$$

Hence, following the same recipe as before, we have

$$u'(c_i) = \rho E_i[\tilde{R}_{i+1} u'(\tilde{c}_{i+1})]$$

New optimisation criterion

- Need to take the FOC w.r.t. alpha as well:

$$0 = E_i[(w_i - c_i)(\tilde{\mu} - 1 - r)V'_{i+1}(\tilde{w}_{i+1})] = E_i[(\tilde{\mu} - 1 - r)V'_{i+1}(\tilde{w}_{i+1})]$$

- The envelope theorem says we can write this as:

$$E_i[\tilde{\mu} u'(\tilde{c}_{i+1})] = (1+r)E_i[u'(\tilde{c}_{i+1})]$$

- We therefore need to solve the following equations in alpha and c:

$$E_i[\tilde{\mu} u'(\tilde{c}_{i+1})] = (1+r)E_i[u'(\tilde{c}_{i+1})]$$

$$u'(c_i) = \rho E_i[\tilde{R}_{i+1} u'(\tilde{c}_{i+1})]$$

Optimal asset-consumption decision

- Now, it’s hard to see this from here, but by starting at the last period and solving backwards, we can show that, if

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

- The asset investment decision is independent of the level of wealth and the time to maturity
- The consumption decision is independent of the investment decision and is a fraction of wealth which depends only on the time left to maturity, i.e. is linear in wealth
- Details in Samuelson’s paper

Merton’s extensions

- Merton developed 2 major extensions of the Samuelson model:

1) A continuous-time model instead of the discrete-time model
2) He allowed for a changing investment opportunity set

- We are going to study both of these, but first we need some continuous-time dynamic programming theory

- This is also called “Optimal Control Theory”.

Simplified Merton (1969)

- Imagine wealth follows the ODE: $\frac{dw_t}{dt} = rw_t - c_t$
- What does this mean in words? c_t
- We want to choose the <u>path</u> of to maximise the following expression:

$$J(w_t, t) = \max_{\{c_t\}} \int_t^T e^{-\rho s} u(c_s) ds$$

- What does this mean in words?
-

Optimal control theory

- We can write our objective function as follows:

Bellman equation

Why?

$$J(w_t,t)=\max_{\{c_t\}} e^{-\rho t}u(c_t)h+J(w_{t+h},t+h)+o(h)$$

$$=\max_{\{c_t\}} e^{-\rho t}u(c_t)h+J(w_t,t)+\frac{\partial J(w_t,t)}{\partial w_t}(w_{t+h}-w_t)+\frac{\partial J(w_t,t)}{\partial t}h+o(h)$$

$$=\max_{\{c_t\}} e^{-\rho t}u(c_t)h+J(w_t,t)+\frac{\partial J(w_t,t)}{\partial w_t}h(rw_t-c_t)+\frac{\partial J(w_t,t)}{\partial t}h+o(h)$$

- Hence

$$0=\max_{\{c_t\}} e^{-\rho t}u(c_t)+\frac{\partial J(w_t,t)}{\partial w_t}(rw_t-c_t)+\frac{\partial J(w_t,t)}{\partial t}$$

First order condition

$$\frac{d}{dc_t}\left[e^{-\rho t}u(c_t)+\frac{\partial J(w_t,t)}{\partial w_t}(rw_t-c_t)+\frac{\partial J(w_t,t)}{\partial t}\right]=0$$

$$e^{-\rho t}u'(c_t)=\frac{\partial J(w_t,t)}{\partial w_t}$$

So if we let $u(c_t)=\dfrac{c_t^{\,1-\gamma}}{1-\gamma}$

$$e^{-\rho t}c_t^{\,-\gamma}=\frac{\partial J(w_t,t)}{\partial w_t},c_t=e^{-\rho t/\gamma}\left[\frac{\partial J(w_t,t)}{\partial w_t}\right]^{-1/\gamma}$$

$$0=e^{-\rho t/\gamma}\left(\frac{\gamma}{1-\gamma}\right)\left[\frac{\partial J(w_t,t)}{\partial w_t}\right]^{(\gamma-1)/\gamma}+\frac{\partial J(w_t,t)}{\partial w_t}rw_t+\frac{\partial J(w_t,t)}{\partial t}$$

Boundary condition

- We need a boundary condition, so let $J(w_T, T) = \int_T^T e^{-\rho s} u(c_s) ds = 0$

- Solution is tricky; we don't need to know the details, but we follow Merton in choosing a trial solution as

$$J(w_t, t) = \frac{b(t)}{1-\gamma} e^{-\rho t} w_t^{1-\gamma}$$

- Substituting the trial solution into our PDE yields the following ODE for $b(t)$, with boundary condition $b(T) = 0$

$$\gamma b(t)^{1-1/\gamma} + ((1-\gamma)r - \rho) b(t) + b'(t) = 0$$

Solution (phew!)

- And the solution to the ODE is:

$$b(t) = \left(\frac{1-e^{\nu(t-T)}}{\nu}\right)^{\gamma}, \quad \nu = \frac{\rho-(1-\gamma)r}{\gamma} = r - \frac{r-\rho}{\gamma}$$

- And the path of optimal consumption is:

$$c_t = w_t\left(\frac{1-e^{\nu(t-T)}}{\nu}\right)^{-1} = \frac{w_t}{\bar{a}^{\nu}_{\overline{t-T|}}}$$

PV at rate r of continuous stream increasing at rate $(r\text{-}\rho)/\gamma$

- This helps us see that consumption must increase exponentially at a rate $\frac{r-\rho}{\gamma}$ so that if $r=\rho$ it stays constant as we had in the discrete case!

Check the solution

- We can check the solution by using the following three equations to derive the same ODE for $b(t)$:

$$\frac{dc_t}{dt} = \frac{(r-\rho)}{\gamma} c_t$$

$$\frac{dw_t}{dt} = rw_t - c_t$$

$$c_t = b(t)^{-1/\gamma} w_t$$

Check of the solution

$$\frac{dc_t}{dt} = \frac{d[b(t)^{-1/\gamma}]}{dt} w_t + b(t)^{-1/\gamma} \frac{dw_t}{dt}$$

$$\frac{(r-\rho)}{\gamma} c_t = \frac{d[b(t)^{-1/\gamma}]}{dt} w_t + b(t)^{-1/\gamma} (rw_t - c_t)$$

$$\frac{(r-\rho)}{\gamma} b(t)^{-1/\gamma} w_t = \frac{d[b(t)^{-1/\gamma}]}{dt} w_t + b(t)^{-1/\gamma} rw_t - b(t)^{-2/\gamma} w_t$$

$$\frac{(r-\rho)}{\gamma} b(t)^{-1/\gamma} = \frac{d[b(t)^{-1/\gamma}]}{dt} + b(t)^{-1/\gamma} r - b(t)^{-2/\gamma}$$

$$\gamma b(t)^{1-1/\gamma} + ((1-\gamma)r - \rho)b(t) + b'(t) = 0$$

Introduce a risky asset

- Luckily, we have already done all the heavy lifting
- We just need to remember our stochastic calculus and all is plain sailing
- The equation governing the development of wealth is now an SDE rather than an ODE:

$$dw_t = w_t((\mu - r)\alpha_t + r)dt - c_t dt + \sigma \alpha_t w_t dz_t$$

- The value function is now maximised over the asset mix and consumption:

$$J(w_t, t) = \max_{\{c_s, \alpha_s\}} E_t \int_t^T e^{-\rho s} u(c_s) ds$$

Stochastic optimal control

- As before, drift on *J* must equal $e^{-rs}u(c(s))$ along the optimal path – Bellman's principle of optimality

Bellman equation

Why?

$$J(w_t,t)=\max_{\{c_t,\alpha_t\}} E_t[e^{-\rho t}u(c_t)h+J(w_{t+h},t+h)+o(h)]$$

$$=\max_{\{c_t,\alpha_t\}} E_t[e^{-\rho t}u(c_t)h+J(w_t,t)+\frac{\partial J(w_t,t)}{\partial w_t}(w_{t+h}-w_t)$$

$$+\frac{1}{2}\frac{\partial^2 J(w_t,t)}{\partial w_t^2}(w_{t+h}-w_t)^2+\frac{\partial J(w_t,t)}{\partial t}h+o(h^2)$$

$$=\max_{\{c_t,\alpha_t\}} e^{-\rho t}u(c_t)h+J(w_t,t)+\frac{\partial J(w_t,t)}{\partial w_t}((\alpha_t(\mu-r)+r)w_t-c_t)h$$

$$+\frac{1}{2}\frac{\partial^2 J(w_t,t)}{\partial w_t^2}\alpha_t^2 w_t^2\sigma^2 h+\frac{\partial J(w_t,t)}{\partial t}h+o(h^2)$$

Merton's 1969 result

$$0 = \max_{\{c_t,\alpha_t\}} e^{-\rho t} u(c_t) + \frac{\partial J(w_t,t)}{\partial w_t}((\alpha_t(\mu - r) + r)w_t - c_t)$$

$$+ \frac{1}{2}\frac{\partial^2 J(w_t,t)}{\partial w_t^2}\alpha_t^2 w_t^2 \sigma^2 + \frac{\partial J(w_t,t)}{\partial t}$$

$$= \max_{\{c_t,\alpha_t\}} e^{-\rho t} u(c_t) + \wp J(w_t,t)$$

Drift operator

(use Ito's Lemma to write out an SDE for J and take the "dt" term which is the drift)

Merton's 1969 result

- In addition we have the first order condition:

$$0 = \frac{d}{dc_t}\left[e^{-\rho t}u(c_t) + \frac{\partial J(w_t,t)}{\partial w_t}((\alpha_t(\mu - r) + r)w_t - c_t) + \frac{1}{2}\frac{\partial^2 J(w_t,t)}{\partial w_t^2}\alpha_t^2 w_t^2 \sigma^2 + \frac{\partial J(w_t,t)}{\partial t} \right]$$

$$= e^{-\rho t}u'(c_t) - \frac{\partial J(w_t,t)}{\partial w_t}$$

- So

$$\frac{\partial J(w_t,t)}{\partial w_t} = e^{-\rho t}u'(c_t)$$

Merton’s 1969 result

- And the first order condition:

$$0 = \frac{d}{d\alpha_t}\left[e^{-\rho t}u(c_t) + \frac{\partial J(w_t,t)}{\partial w_t}((\alpha_t(\mu - r) + r)w_t - c_t) + \frac{1}{2}\frac{\partial^2 J(w_t,t)}{\partial w_t^2}\alpha_t^2 w_t^2 \sigma^2 + \frac{\partial J(w_t,t)}{\partial t} \right]$$

$$= \frac{\partial J(w_t,t)}{\partial w_t}(\mu - r)w_t + \frac{\partial^2 J(w_t,t)}{\partial w_t^2}\alpha_t w_t^2 \sigma^2$$

- So:

$$\alpha_t = -\frac{\mu - r}{w_t \sigma^2}\frac{\frac{\partial J(w_t,t)}{\partial w_t}}{\frac{\partial^2 J(w_t,t)}{\partial w_t^2}}$$

Substituting F.O.C.s into Bellman equation

$$0 = \frac{\gamma}{1-\gamma}\left(\frac{\partial J(w_t,t)}{\partial w_t}\right)^{(1-\gamma)/\gamma} e^{-\rho t/\gamma} + \frac{\partial J(w_t,t)}{\partial t}$$

$$+\frac{\partial J(w_t,t)}{\partial w_t} r w_t - \frac{(\alpha - r)^2}{\sigma^2 w_t} \frac{\left(\frac{\partial J(w_t,t)}{\partial w_t}\right)^2}{\frac{\partial^2 J(w_t,t)}{\partial w_t^2}}$$

First order conditions

$$\frac{\partial J(w_t,t)}{\partial w_t} = e^{-\rho t} u'(c_t)$$

$$\alpha_t = -\frac{\mu - r}{w_t \sigma^2} \frac{\frac{\partial J(w_t,t)}{\partial w_t}}{\frac{\partial^2 J(w_t,t)}{\partial w_t^2}}$$

Called the Hamilton-Jacobi-Bellman (HJB) equation

$$J(w_T,T) = 0$$

Terminal boundary condition

Solution of the system of equations

- This looks impenetrable, so what do we do?
- Luckily, the same trial solution

$$J(w_t, t) = \frac{b(t)}{1-\gamma} e^{-\rho t} w_t^{1-\gamma}$$

- works, but yields a slightly different differential equation for $b(t)$:

$$\gamma b(t)^{1-1/\gamma} + ((1-\gamma)(r + \frac{(\alpha - r)^2}{2\sigma^2\gamma}) - \rho) b(t) + b'(t) = 0$$

- with a similar solution:

$$b(t) = (\frac{1-e^{\nu(t-T)}}{\nu})^{\gamma}, \quad \nu = \frac{\rho - (1-\gamma)(r + \frac{(\mu - r)^2}{2\sigma^2\gamma})}{\gamma}$$

Optimal policy variables

- Optimal consumption (from FOC for consumption)

$$c_t = \frac{w_t}{\frac{1 - e^{\nu(t-T)}}{\nu}} = \frac{w_t}{\bar{a}^{\nu}_{\overline{T-t}|}}$$

- Optimal investment policy (from FOC for asset allocation)

$$\alpha_t = \frac{\mu - r}{\gamma \sigma^2}$$

Excess instantaneous expected return on risky asset

Coefficient of relative risk aversion

Variance of return on risky asset

What *doesn't* affect policy variables?

- Optimal consumption
  - Linear in wealth
- Optimal investment policy
  - Unaffected by amount of wealth
  - Unaffected by time to horizon

- Results are identical to Samuelson's results in discrete time

One more thing

- Now that we know that $a_t = \frac{m- \ r}{gs^2}$, we can rewrite:

$$\nu = \frac{\rho - (1-\gamma)(r + \frac{(\mu - r)^2}{2\sigma^2\gamma})}{\gamma} = r + (\mu - r)\alpha_t - \frac{r - \rho + \frac{1}{2}(1+\gamma)(\mu - r)\alpha_t}{\gamma}$$

- Hence $c_t = \frac{w_t}{\bar{a}^{\nu}_{\overline{T-t}|}}$

PV at expected return on assets of a consumption stream starting at 1 p.a. and increasing continuously at rate $\frac{r - \rho + \frac{1}{2}(1+\gamma)(\mu - r)\alpha_t}{\gamma}$

Significance of Merton's result

- The result exposes the "time diversification" fallacy of investment very nicely (in the case of risky investment returns following a BM)
- If you could diversify across time, as the length of time increased, you would increase your holdings in the risky asset (because it would become less risky to you as your time horizon moved further away)
- This doesn't happen
- So why can't you diversify across time like you can across (investment) space?

A simple extension

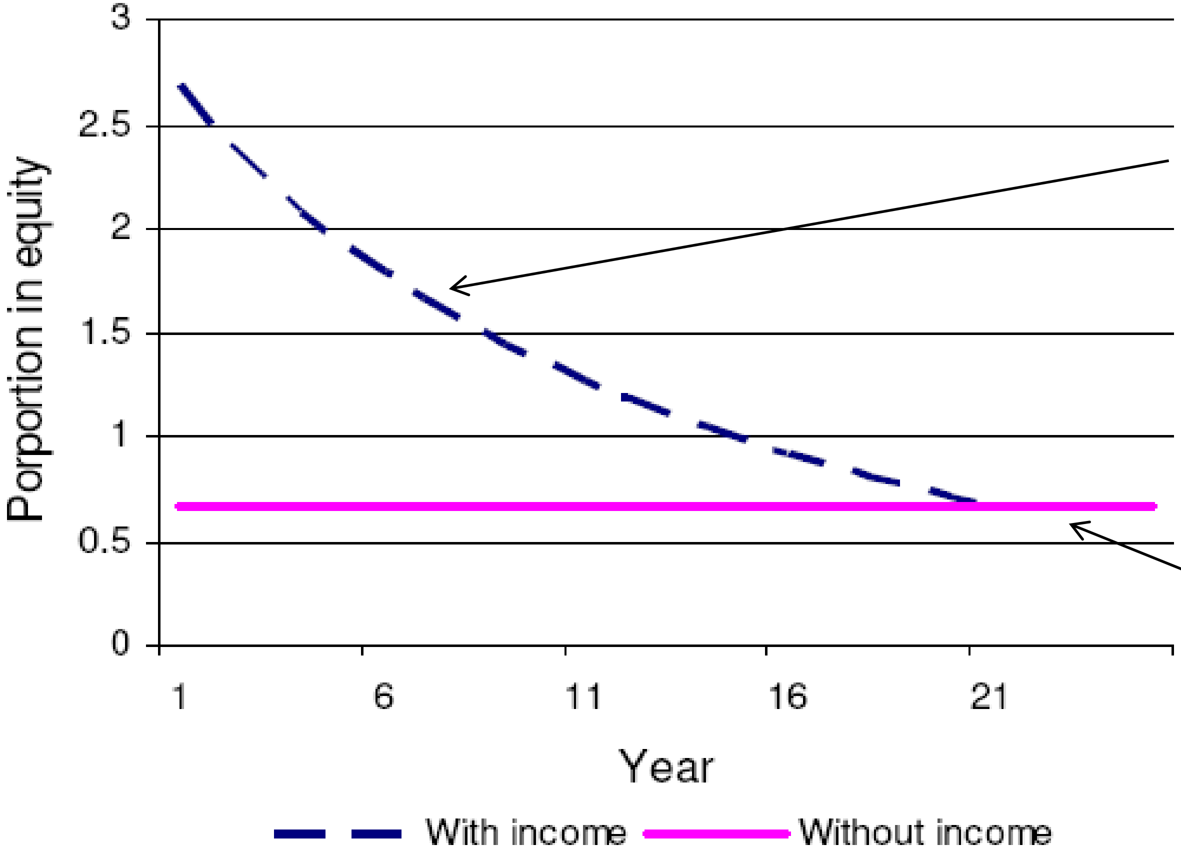
- Most people earn labour income rather than consume out of a fixed endowment
- How would Merton's result change then?
- Wealth now follows this SDE:

$$dw_t = (a_t(m - r) + r)w_t dt + y_t dt + s a_t w_t dz_t - c_t dt$$

- Individual's optimal behaviour is to "capitalise" the certain income flow at the risk-free rate, and invest the augmented portfolio in exactly the same way as the portfolio in the case of no wage income
- For proof, see Merton (1971) but the basic intuition is clear (I hope)

Merton, with labour income

- So what does this mean we would observe?



A systematic age-related reduction in the proportion of the portfolio invested in the risky asset

Once there is no more income the effect disappears and the Merton model returns

Another simple extension

- In practice there are many risky assets (n), not just one
- Answer should be fairly obvious, but if we have wealth governed by the following SDE:

$$dw_t = w_t((\boldsymbol{\mu} - r\mathbf{1})^{\mathrm{T}}\boldsymbol{\alpha}_t + r)dt - c_t dt + \boldsymbol{\alpha}_t^{\mathrm{T}}\boldsymbol{\Omega} w_t d\mathbf{z}_t$$

- where $\boldsymbol{\mu} =$ (n x 1) vector of expected returns on risky assets
- $\mathbf{1} =$ (n x 1) vector of ones
- $\boldsymbol{\alpha}_t =$ (n x 1) vector of asset allocation to each risky asset
- $\boldsymbol{\Omega} =$ (n x d) matrix of sensitivities of stock prices to BM's
- $\mathbf{z}_t =$ (d x 1) vector of d independent Brownian motions

Many risky assets: the one fund theorem

- Then the optimal investment strategy is

$$\boldsymbol{\alpha}_t = \frac{(\boldsymbol{\Omega}\boldsymbol{\Omega}^{\mathrm{T}})^{-1}(\boldsymbol{\mu} - r\mathbf{1})}{g}$$

- So invest a constant proportion of your wealth in each of the risky assets according to their variance-covariance structure and the remainder in the riskless asset
- As your risk aversion changes, reduce your holdings in each of the risky assets proportionally
- Called a “one fund theorem” (have you seen one of these before?)

The two-fund theorem

- Given log-normal asset returns, and investors with ANY set of preferences, time horizon or wealth, only two mutual funds need to be provided to the market to satisfy the investment needs of ALL investors
- This is a generalisation of the one-fund theorem (where investors were assumed to have CRRA preferences)
- Does this make sense from the point of view of Markowitz analysis?
- How does it differ?

The three-fund theorem

- This examines the case of changing investment opportunities
- Introduces the concept of "intertemporal hedging demand":
  - You hold a portfolio of assets because it hedges you against future changes in investment opportunities
- Three funds are:
  - Market portfolio
  - Riskless asset
  - "Maximally correlated portfolio"
- Details in Merton (1990, Chapter 11)

Asset allocation models in practice

- We are now going to discuss these asset allocation models in practice
- In order to assess how accurate they are, we need to first understand something about the actual portfolios owned by households

Household portfolios

- Household portfolios consist of all the assets that households own
- Why do we concentrate on households and not on individuals?
- What are the main assets that households own?

Main household assets

- House
- Car & other durable assets
- Bank account
- Private pension
- Mutual funds
- Stocks
- State pension
- Other state benefits (unemployment, health care etc)

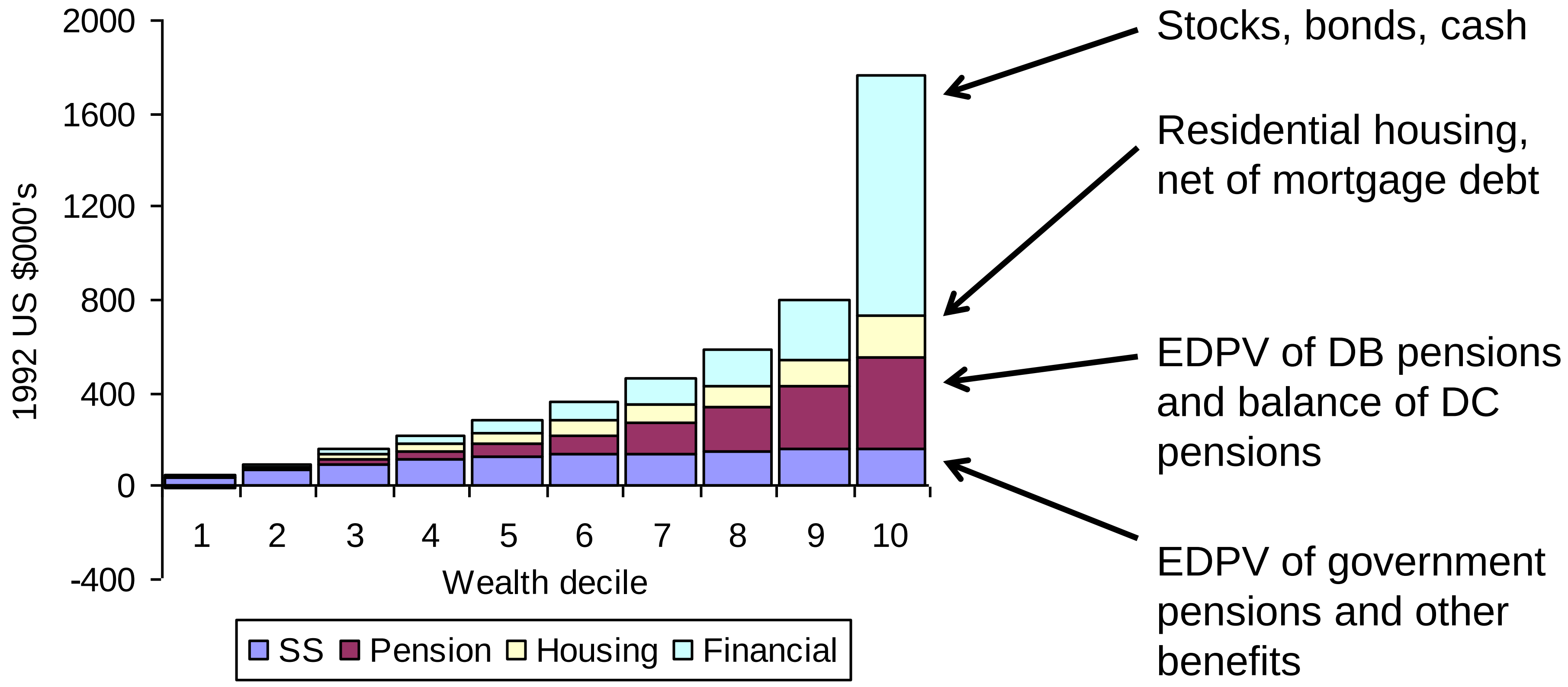
How important are each of these assets?

- It is very difficult to say
- Household portfolios differ systematically depending on various factors, including age and wealth
- So let’s study three stylised facts about household portfolios which hold across the OECD to provide some answers

Main surveys used to measure HH portfolios

- Survey of Consumer Finances (SCF), US dataset
- Panel Study of Income Dynamics (PSID), US dataset
- Health and Retirement Study (HRS), US dataset
- Family Resources Survey (FRS), UK dataset
- British Household Panel Survey (BHPS), UK dataset
- English Longitudinal Study of Ageing (ELSA), UK dataset
  - Only HRS and ELSA include measures of private and state pensions in anything like accurate form

A: How household portfolios change with wealth



- Portfolios of 56 year-olds in the US in 1992 (HRS)

A: How household portfolios change with wealth

- Wide disparity in size of portfolio (USA data, but broadly true everywhere)
  - Poorest 10% own very little & what they do is mostly in the form of social security
  - Richest 10% own much more & it is mostly in financial assets
- The median holdings in the US
  - 40% in social security
  - 20% in pensions
  - 20% in housing
  - 20% in financial wealth

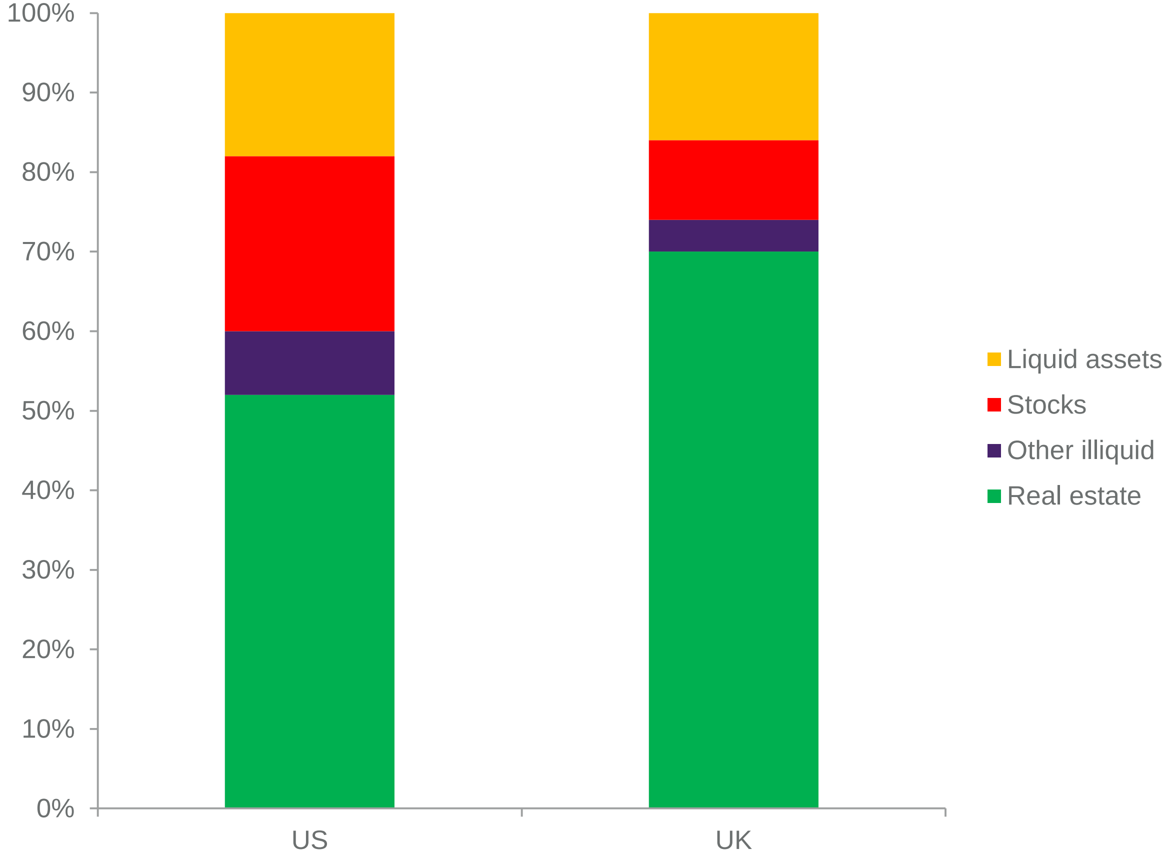
} These make up the majority of household wealth for a large number of US households

A: How household portfolios change with wealth

- Wide disparity in compositions
- Pension wealth is unequally distributed
- Financial wealth is more unequally distributed.

- How would this picture change in the UK?
- Overall, it is similar
- Sate pension wealth is less
- Possibly less inequality
- More invested in housing & less in financial wealth

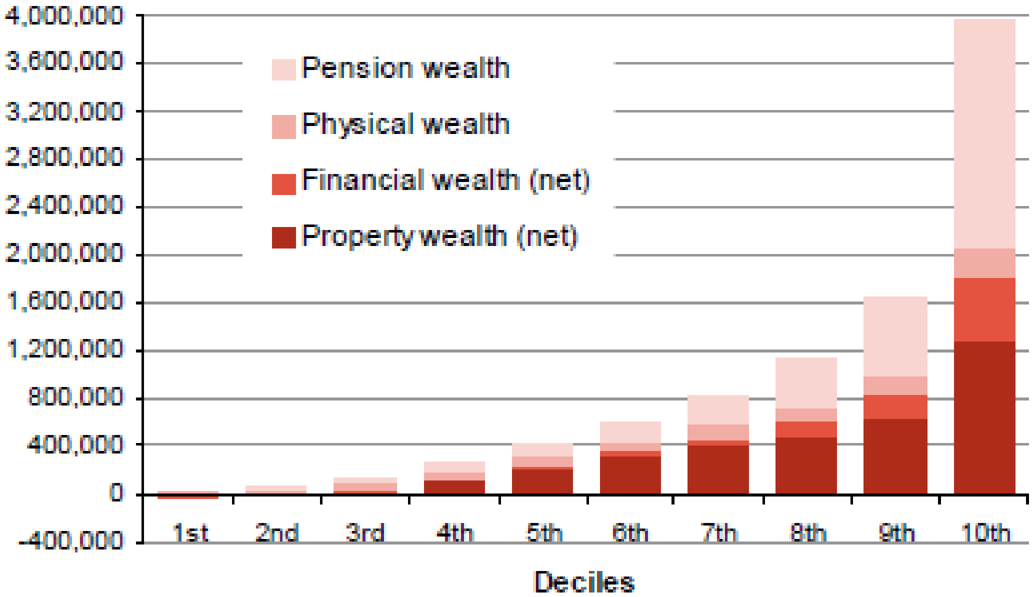
US & UK non-pension portfolios



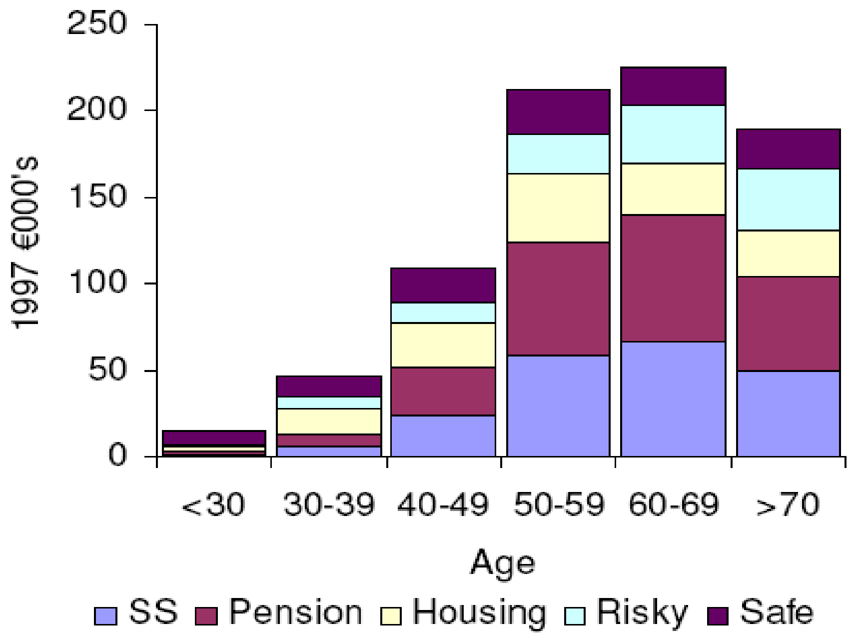
Source: Banks et al (1994)

Wealth and asset survey (2006/2008)

- Total HH wealth ~£9trn (40% housing, 40% pensions, 10% financial, 10% other)
- Median HH wealth £204,500, Gini coefficient = 0.61

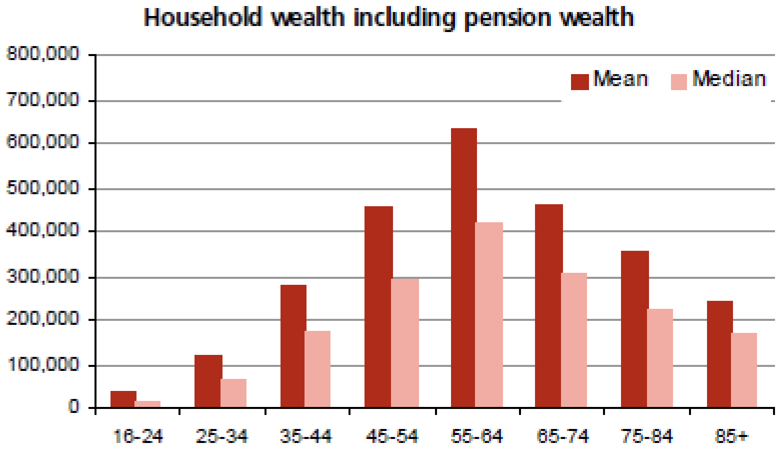


B: How household portfolios change with age



Remember that these averages mask significant heterogeneity by wealth at each age, as shown in the previous slide

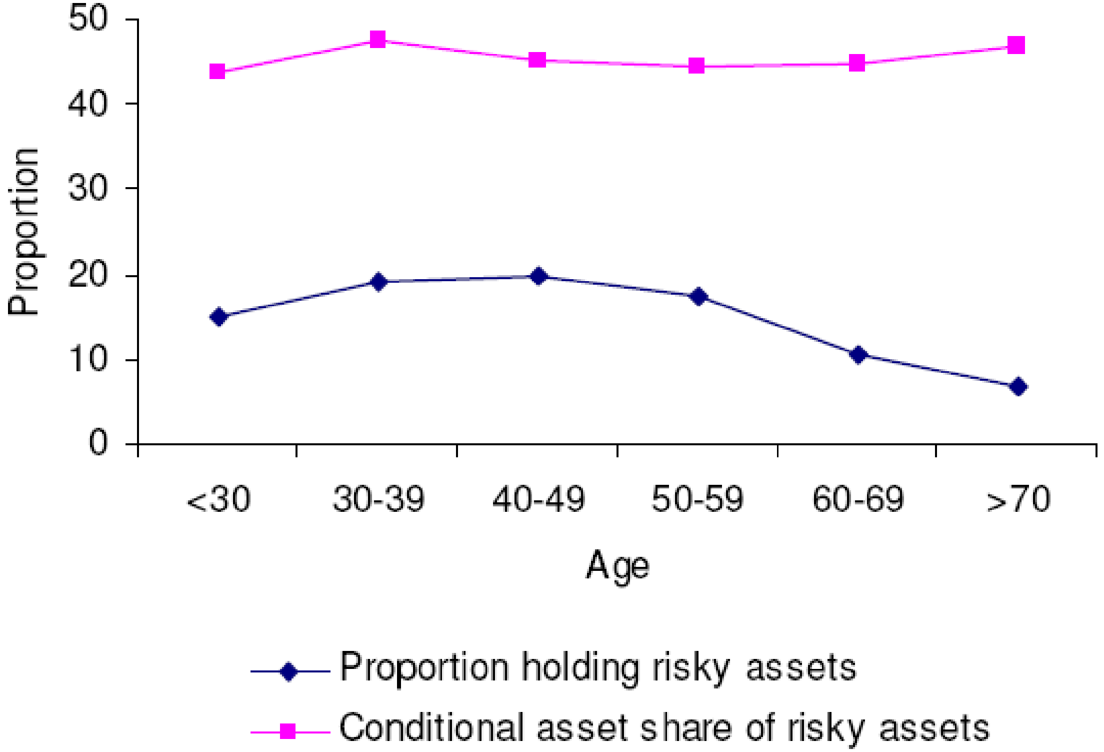
UK wealth and asset survey



B: How household portfolios change with age

- Younger people invest mainly in safe assets, first risky asset is (usually) the house they live in
- Once the house has been purchased, they began to accumulate other risky financial assets
- Rapid wealth accumulation between 35-55
- Gradual decumulation after retirement
  - Hard to distinguish between cohort effects and decumulation: this problem plagues all analysis)
- Social security pensions extremely important, particularly to those near retirement
- 40/20/20/20 more like 30/30/20/20 in the Netherlands

C: Substantial heterogeneity in holding of risky assets in population



- Observed proportion of population holding risky assets and conditional share of assets in risky assets in Italy

C: Substantial heterogeneity in holding of risky assets in population

- However:
- Share of risky assets, conditional on holding them is roughly constant for all ages
- Proportion of individuals holding any risky assets changes
  - First increases
  - Then decreases
- Proportion typically quite low (in Italy ~ 15% & US/UK ~ 50%), although apparently quite high in Sweden
- Important to distinguish between direct and indirect holdings

Aside on pensions

•When we think about state and occupational pensions in the context of our three stylised facts, for whom are they most important?

- State pensions are extremely important for the poor.
- Occupational pensions are most important for those in the middle of the wealth distribution
- Together, these represent the most significant accumulation of wealth for individuals in these groups ???
- Why?

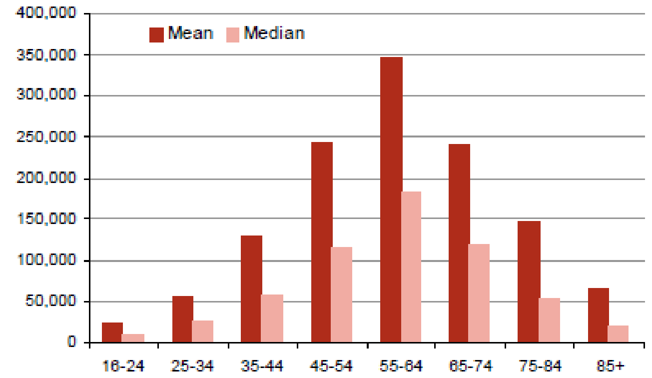
Pensions: wealth and asset survey

Pension wealth more unequally distributed than total wealth (Gini coefficient = 0.77)

Distribution of household private pension wealth[1]: by age of household head, 2006/08

Great Britain

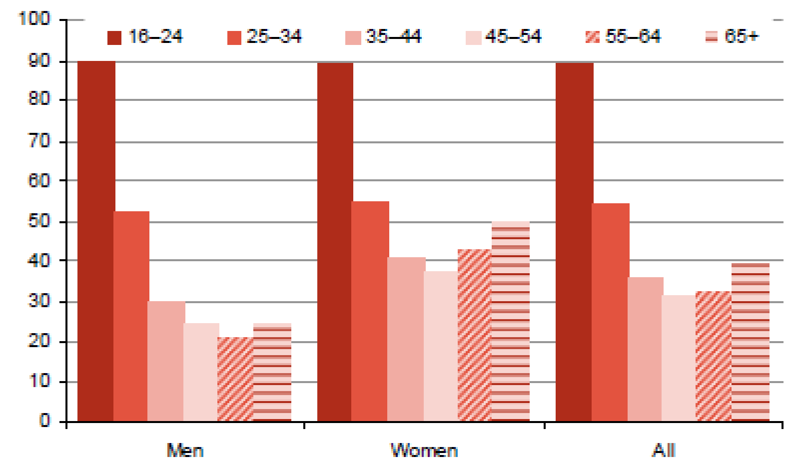
£



Proportion of individuals with no private pension wealth: by age and sex, 2006/08

Great Britain

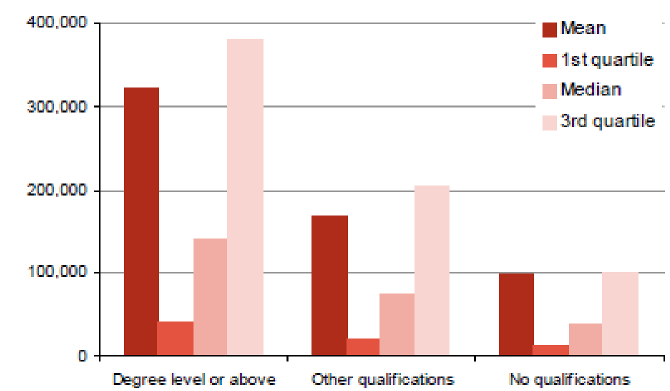
Percentages



Distribution of household private pension wealth[1]: by education of household head, 2006/08

Great Britain

£



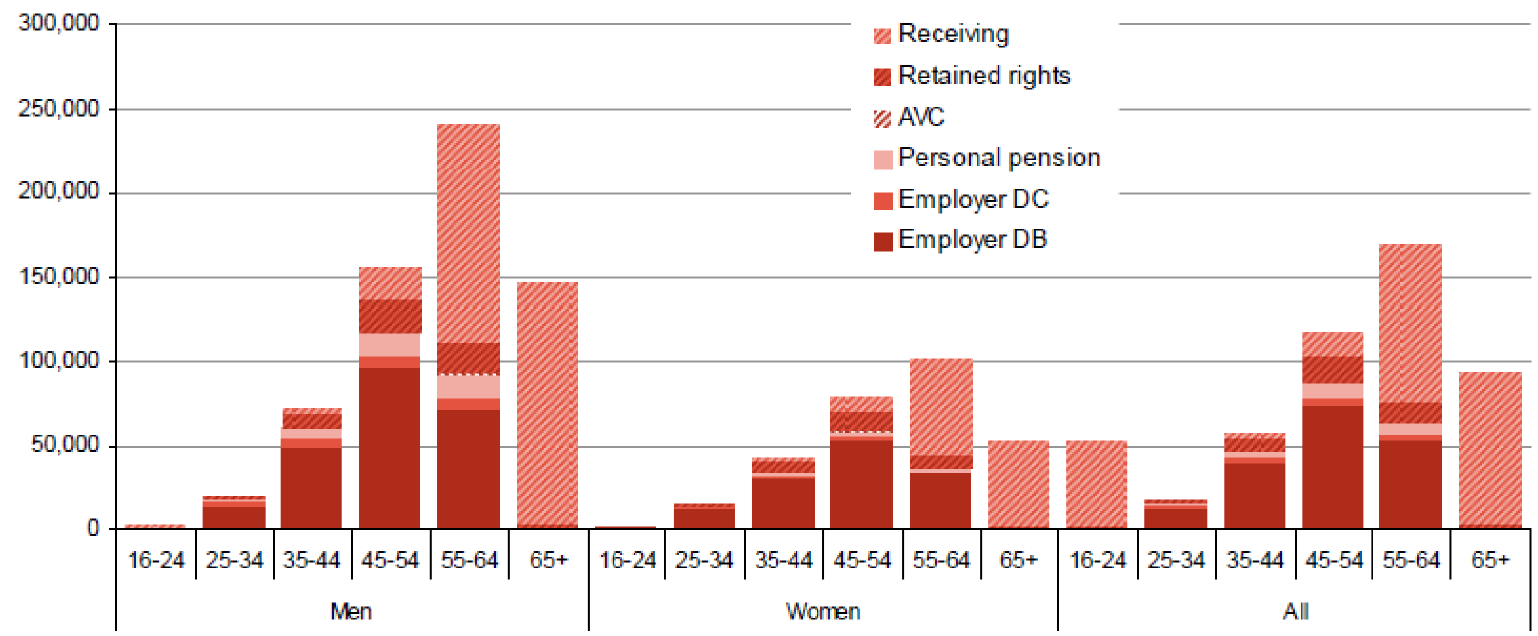
1 Excludes those with zero pension wealth.

Pensions: wealth and asset survey

Mean wealth held by individuals in private pensions[1]: by type of pension, age and sex, 2006/08

Great Britain

£



Aside on housing

- Housing is a risky asset
- It constitutes the majority of most people's non-state pension wealth
- Many people hold highly leveraged positions in housing, particularly when they are young
- Housing as an asset class has very special features:
  - It is both a consumption asset and an investment
  - It is extremely undiversified
  - Transactions costs are high
  - You can choose to buy or rent

Merton model and our three facts

- How do you think the Merton model performs when viewed against our three facts?

- Fact A: Poorer household hold very few financial assets and when they do they are largely held in safe assets
- Fact B: Younger household hold safer financial assets than older households
- Fact C: Proportion of portfolio held in risky assets is stable across life; the probability of holding risky assets changes dramatically

Merton-Samuelson asset allocations

- How well does the Merton-Samuelson model perform?

	Germany	Italy	Japan	Netherl.	UK	US
Period	1/1950-12/1996	12/1928-12/1996	4/1949-12/1996	1/1921-12/1996	1/1921-12/1996	1/1921-12/1996
Real return	7.6%	3.2%	7.2%	2.8%	3.6%	5.5%
Volatility	2.4%	6.6%	3.6%	2.2%	2.5%	2.5%
Equity portfolio share ($\gamma = 3$)	104%	16%	67%	42%	49%	73%
Equity portfolio share ($\gamma = 5$)	62%	10%	40%	25%	29%	44%

Source: Author's calculations derived using parameter values from Jorion and Goetzmann (2000). Returns and volatilities measured in local currency in real terms.

- Merton-Samuelson asset allocation for 6 OECD countries

Response of financial economists?

- Merton & Samuelson's work is over 30 years old (both have since won the Nobel prize)
- So a generation of financial economists has been able to work with these models and extend them
- Extensions fall into three areas
  - Identifying the problems
  - Theoretical tools to enable easier & more general solutions
  - Extensions of the models to take into account the two above points

Equity risk premium puzzle

- Merton-type models predict that individuals hold too many equities relative to what they actually hold, given actual levels of risk aversion and actual historical equity returns and risks
- Alternatively, equities seem to be a very good investment given their risk and return properties and what we know about preferences and investment theory
- This was first formulated by Mehra and Prescott (1985) as “The Equity Risk Premium Puzzle”
- The puzzle has a portfolio theory counterpart

Equity risk premium puzzle

- Return on equities are too high relative to their volatility & what
- we know about individual risk aversion

- Agents in Merton type models seem to want to hold much more
- equity then they do in actuality

Explanations of ERP puzzle

- There are many explanations of the equity risk premium puzzle, and many of these have been used to explain the apparent failure of Merton-type models, too
  - Durable goods (esp housing)
  - Transactions costs (only part of the population invests in the risky asset)
  - Behavioural finance
  - Habit formation models

These last two are equivalent to changing the preference structure that Merton assumed

Practical extensions of Merton-type models

- Incomplete markets
  - In a Merton-type world, all risks can be hedged by holding portfolios of traded assets
  - This is called “complete markets”
  - However, in actual fact, markets are incomplete
    - Transaction costs
    - Taxes
    - Untraded assets
    - Liquidity constraints
  - People have built life-cycle portfolio models incorporating these elements

Practical extensions of Merton-type models

- Housing (durable goods)
  - Housing is an asset with very special characteristics and it is the most important asset most people own
  - Lifecycle models incorporating housing are difficult to solve but there are more and more of them

Practical extensions of Merton-type models

- Real options
  - Individuals have some real options that they can use to mitigate risk
    - Most importantly, the option to work more if asset markets do badly
    - This actually makes the Merton problem worse, not better

Practical extensions of Merton-type models

- Changing the preference structure
  - Habit formation type models
    - Investors have a level of consumption they get used to
    - This is equivalent to assuming very high risk aversion
  - Behavioural finance
    - We’ll cover this in a later lecture

Theoretical extensions

- Cox-Huang methodology
  - There is an easier (?) way to solve the Merton-type dynamic program & more complex problems in incomplete markets
  - Uses the principles of replicating portfolios and martingales to replace the dynamic problem with a static problem
  - Basic references are Cox and Huang (1989, JET), and Merton (1990, Chapter 6)

Conclusion

- We extended our lifecycle models to include asset allocation in discrete time
- We spent a lot of time on the fundamental model: Merton’s asset allocation model in continuous time
- We examined the implications of the model for the time diversification fallacy
- We learned one, two and three fund theorems
- We studied some stylised facts about household portfolios and learned that they don’t square well with the theoretical models
- We learned about some extensions to the theory